A Quantification Model for Crack Propagation of R/C Members under Earthquake Loading

N. Takahashi and Y. Nakano
The University of Tokyo, Japan

Y. Ito
Asahi Building-Wall Co., Ltd., Japan

ABSTRACT:
To evaluate visible damage of reinforced concrete (R/C) members such as crack width and length, cyclic and monotonic load tests of scaled R/C members were carried out. Firstly, a predictive model is proposed to quantify the propagation of crack length. This model enhanced the mechanical model which was proposed in CEB-FIP code 1978. Secondly, a predictive model is proposed to quantify the crack width. The model consists primarily of a geometrical model considering the relationship between the sum of crack widths and drift ratio. Finally, a predictive model is proposed to quantify crack length distribution to crack width. The model consists primarily of a probabilistic model between crack widths and lengths. These proposed models show that the estimated flexure crack widths and length approximate the measured crack widths and length, but the estimated shear crack widths and length disagree with test results.

Keywords: Seismic Reparability Performance, Quantifying the Visible Damage, Reinforced Concrete Buildings

1. INTRODUCTION

To evaluate visible damage of reinforced concrete (R/C) members such as crack width and length, cyclic load tests of 1/3 scaled R/C members (load test series 2008) and monotonic load tests of 1/2 (load test series 2011) and were carried out. Then damage propagation process represented by crack width and length was observed in these tests.

Firstly, based on the 1/2 scaled R/C members tests named “load test series 2011,” a predictive model is proposed to quantify the propagation of crack length. This model enhanced the mechanical model which was proposed in CEB-FIP code 1978 (CEB-FIP, 1978). The CEB-FIP code considered the crack propagation under a service load level. The proposed model in this paper considers the crack propagation under not only a service load but a seismic load level.

Secondly, based on the 1/3 scaled R/C members tests named “load test series 2008,” a predictive model is proposed to quantify the propagation of crack width. The model consists primarily of a geometrical model considering the relationship between the sum of crack widths and drift ratio. It can take account of the crack propagation process due to the current drift ratio; crack widths can be estimated corresponding to the residual drift through the geometrical model. It can also take account of the difference in crack widths at peak load stages and those at unloaded stages.

Finally, a predictive model is proposed to quantify the relationship between crack width and corresponding crack length. The model consists primarily of a probabilistic model between crack widths and lengths. These proposed models show that the estimated flexure crack widths and length successfully approximate the measured crack widths and length, but the estimated shear crack widths and length disagree with test results.

2. EXPERIMENTAL PROGRAM

2.1. Load Test Series 2011

2.1.1. Test specimens, setup and instrumentation

Three R/C beam specimens proportioned to approximately 1/2 of full scale were tested under monotonic loading. The design parameters and corresponding values are given in Table 1. The dimension for the test specimens and test setup are shown in Fig. 1, 2 and 3. Crack widths were measured at the points shown in Fig. 4. Crack lengths were measured by image processing of sketched cracking pattern.

Table 1. Description of test specimens

Specimen	Concrete Strength [N/mm^2]	Rebar / Tensile reinforcement ratio to the section	Yield strength of rebar [N/mm^2]	Lateral reinforcement / Lateral reinforcement ratio to the section	Yield strength of lateral reinforcement [N/mm^2]	Failure mode
F-60	30.5	8-D13 / 0.0067	413 (SD295)	D6@60 / 0.0049	418 (SD295)	Flexure
F-90	32.0			D6@90 / 0.0033	387 (SD345)	Flexure
FS-90	32.5	8-D16 / 0.0104	569 (SD490)	ϕ9@90 / 0.0066	358 (SR235)	Flexure -Shear

D: diameter of deformed bar

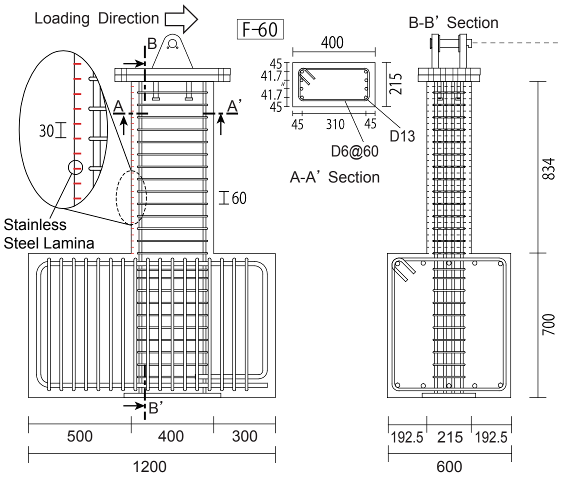


Figure 1. Dimension of beam specimen (F-60)

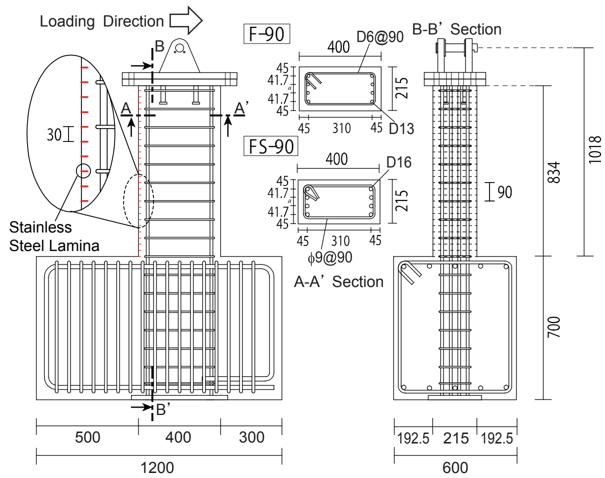


Figure 2. Dimension of beam specimen (F-90/FS-90)

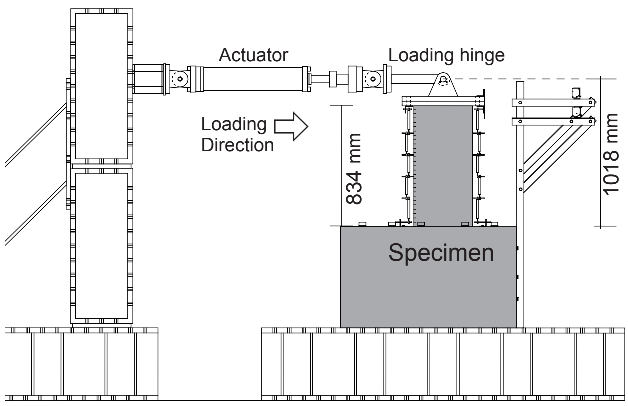


Figure 3. Test setup

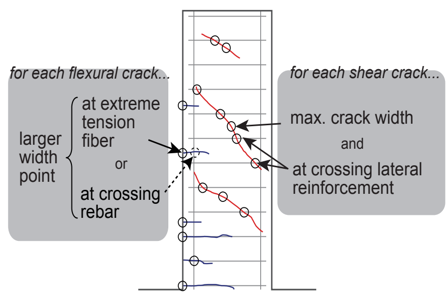


Figure 4. Crack measurement point

2.1.2. Test results

Fig. 5shows the shear force versus drift response for each specimen and the cracking pattern at 4.0% drift. Measured maximum crack widths are shown in Fig. 6. Measured crack lengths are shown in Fig. 7. All specimens opened existing flexural cracks due to increase in drift ratio instead of generating new flexural cracks after yielding. Therefore flexural crack widths were increased but flexural crack lengths did not increase after yielding. Specimen F-60 and F-90 designed to fail in flexure generated shear cracks after yielding and their length were the same as their flexural crack length. Specimen FS-90 designed to fail in flexural-shear generated shear cracks after yielding and its length was twice as long as its flexural crack length.

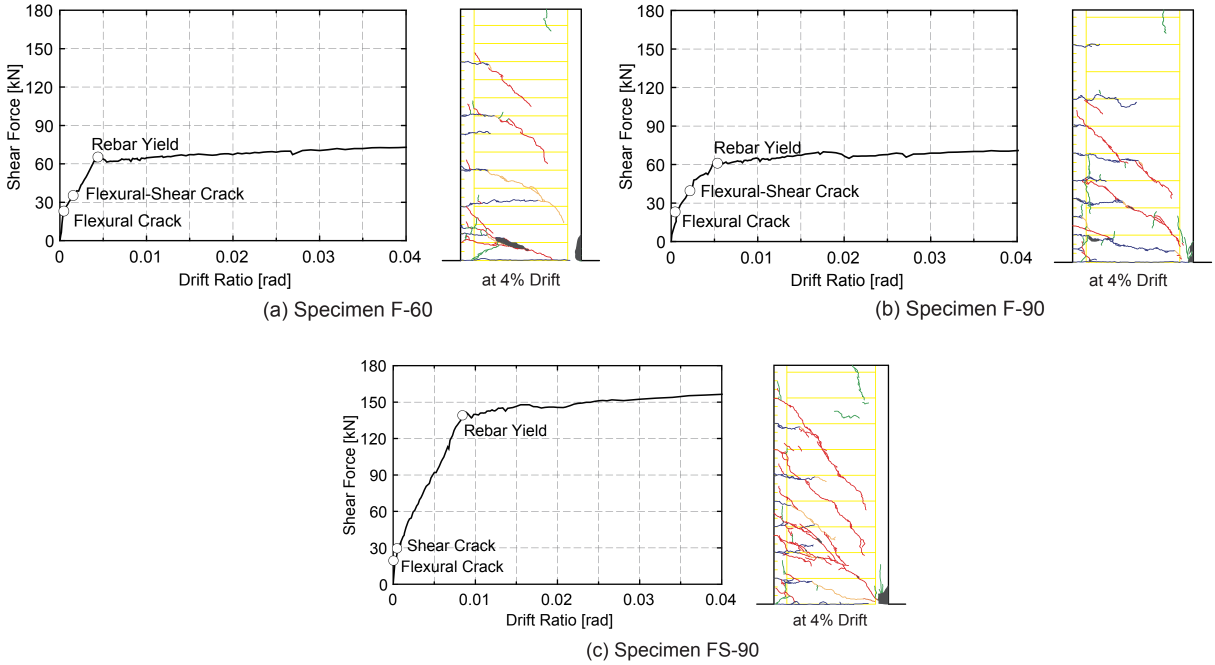


Figure 5. Shear force versus drift ratio response, and cracking pattern

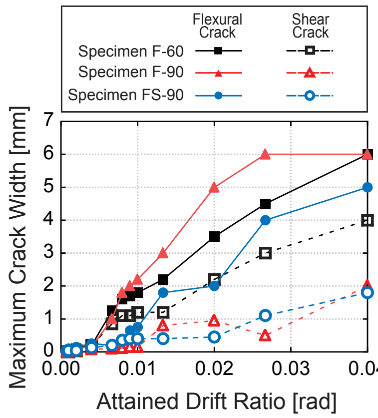


Figure 6. Crack width for attained drift ratio

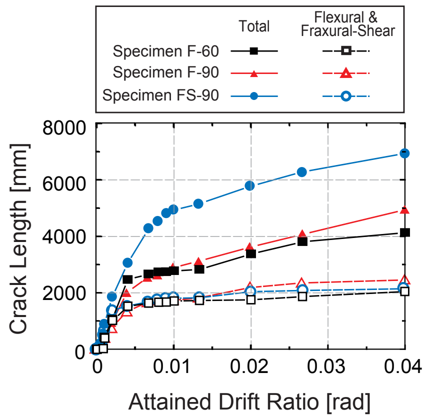


Figure 7. Crack length for attained drift ratio

2.2. Load Test Series 2008

2.2.1. Test specimens, setup and instrumentation

Two R/C beam specimens proportioned to approximately 1/3 of full scale were tested under cyclic loading. The design parameters and corresponding values are given in Table 2. The dimension for the test specimens and test setup are shown in Fig. 8 and 9. To obtain the propagation of crack width and length corresponding to attained and present drift ratio, the cyclic displacement pattern shown in Fig.

10 was operated. Crack widths were measured at the points shown in Fig. 4. Crack lengths were measured by image processing of sketched cracking pattern.

Table 2. Description of test specimens

Specimen	Concrete Strength [N/mm^2]	Rebar / Tensile reinforcement ratio to the section	Yield strength of rebar [N/mm^2]	Lateral reinforcement / Lateral reinforcement ratio to the section	Yield strength of lateral reinforcement (N/mm^2)	Failure mode
F-1	35.4	8-D13 / 0.0121	351 (SD295)	D4@60 / 0.0022	385 (SD295)	Flexure
S-1	25.9		902 (KSS785)		385 (SD295)	Shear

D: diameter of deformed bar

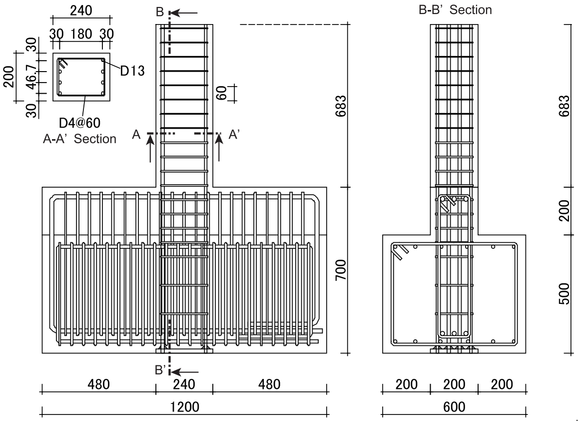


Figure 8. Dimension of beam specimen

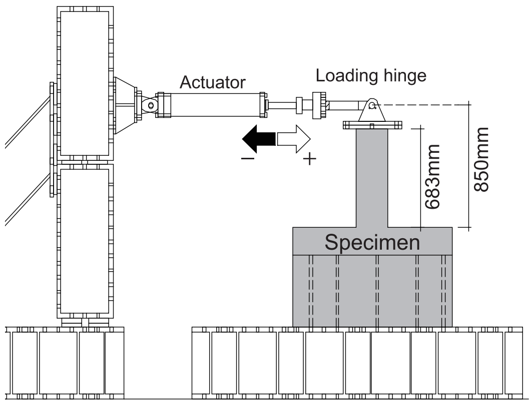


Figure 9. Test setup

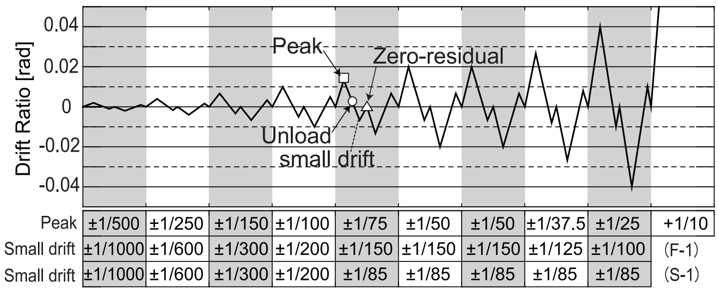


Figure 10. Cyclic displacement pattern

2.2.2. Test results

Fig. 11 shows the shear force versus drift response for each specimen and the cracking pattern at 4.0% drift. Measured maximum crack widths are shown in Fig. 12. Measured crack lengths are shown in Fig. 13. Specimen F-1 designed to fail in flexure opened existing cracks due to increase in drift ratio instead of generating new cracks after yielding. Therefore total crack length did not increase significantly. On the other hand, Specimen S-1 designed to fail in shear generated new cracks due to the increase in drift ratio after yielding. Crack length as well as crack width increased. Crack width and length of specimen S-1 increased rather than specimen F-1 in large drift.

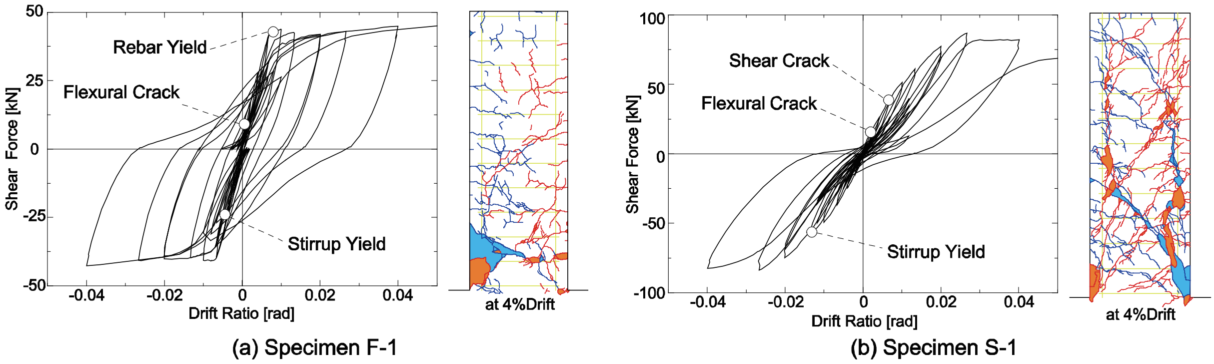


Figure 11. Shear force versus drift ratio response, and cracking pattern

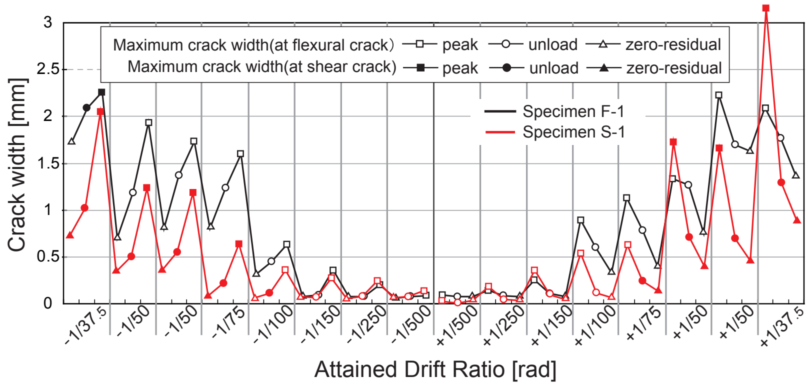


Figure 12. Crack width for attained drift ratio

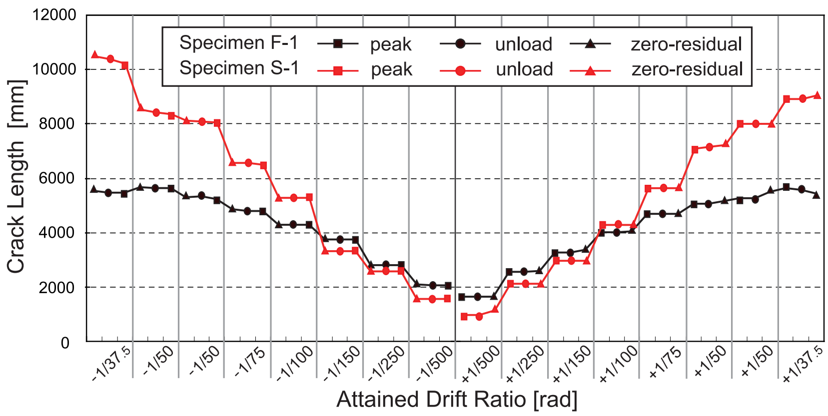


Figure 13. Crack length for attained drift ratio

3. DAMAGE ESTIMATION

3.1. Crack length eastimation model

In this paper, flexural-shear cracks are modelled as bilinear according to the crack growth angle as shown in Fig. 14. Flexural and flexural-shear cracks are estimated based on a fiber model analysis. Kent & Park model (Kent & Park, 1971), Okamura & Maekawa model (Okamura & Maekawa, 1991), and bilinear model are employed to the compressive concrete, the tensile concrete, and the reinforcing bar model, respectively. On the other hand, shear cracks are estimated from a stabilized crack pattern after shear cracking strength, where doesn’t consider the propagation of shear crack length. The following paragraphs show the detailed process and the example of crack length estimation.

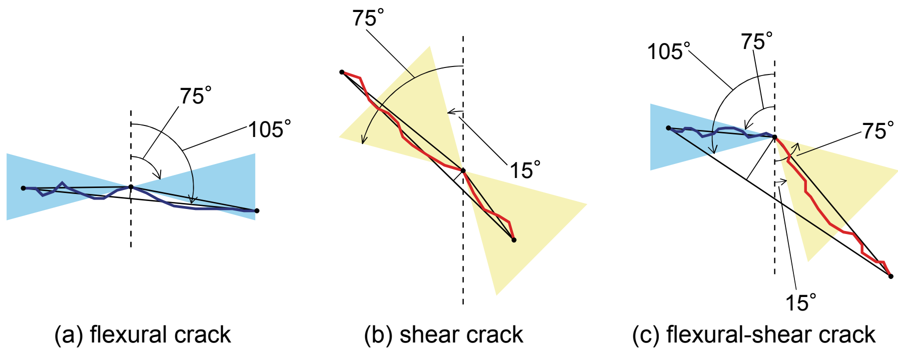


Figure 14. Crack type definition due to its growth angle

3.1.1. Propagation of flexural crack

Flexural cracks generate at the extreme tension fiber where the moment M is larger than flexural cracking moment M_c. This cracking zone, which length is defined as l_{cr}, is expressed as Eqn. 3.1.

$$l_{cr} = (1 - \frac{M_c}{M})H \tag{3.1}$$

where, H: shear span. The number of flexural cracks is also expressed as Eqn. 3.2.

$$n = \frac{l_{cr}}{S_{av}} + 1 \tag{3.2}$$

where, S_{av}: Average flexural cracking space (Morita, 1969). The length of flexural crack is defined as the distance from the extreme tension fiber to the point of concrete tensile strength estimated from a fiber model analysis.

3.1.2. Propagation of flexural-shear crack after the inflection point

To estimate the propagation of flexural-shear cracks after the flexural cracks extending, the inflection points (X_b, Y_b) of flexural-shear cracks are defined shown in Fig. 15. At the inflection point, the angle of principal stress to the axis of the beam calculated from Mohr's stress circle comes under 75 degrees. The crack over the inflection point will propagate to the targets shown in Fig. 16 according to their inflection point coordinates. When an inflection point is included in plastic hinge area, the crack is oriented to the stirrup at critical section in compressive zone. When an inflection point is included in the area adjoining a plastic hinge area where is expressed as Eqn. 3.3, the crack is propagate with a constant degrees ϕ to the axis of the beam.

$$Y_{bs} \leqq (j_e \cot\phi + S_{m\theta}) - X_{bs} / \tan\phi) \tag{3.3}$$

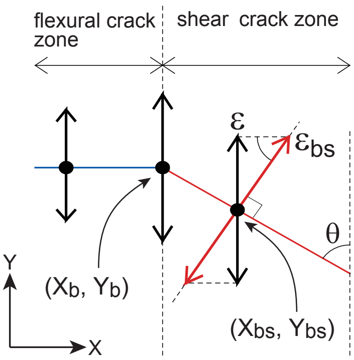


Figure 15. Inflection point and converted strain ε_{bs}

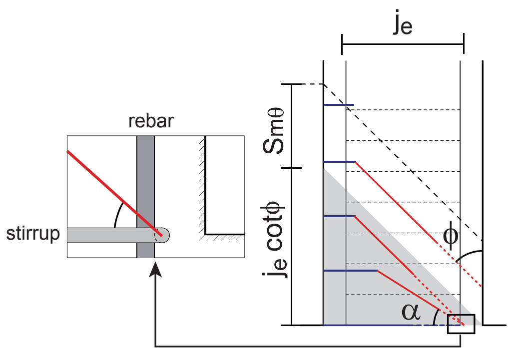


Figure 16. Orientation of crack propagation

where, j_e: the distance of stirrup in loading direction, $S_{m\theta}$: average shear cracking space (AIJ 2004), respectively. The length of flexural-shear crack is defined as the sum of the distance from the extreme tension fiber to the inflection point and the distance from the inflection point to the point of converted concrete tensile strength estimated from a fiber model analysis. That is the point where the converted strain ε_{bs}, which is expressed as Eqn. 3.4, is larger than the strain of concrete tensile strength ε_{ct}.

$$\varepsilon_{bs} = \varepsilon / \sin\theta \tag{3.4}$$

3.1.3. Shear crack length model

When the shear force is larger than shear cracking strength, shear cracks are generated in the area without the plastic hinge area and the area adjoining the plastic hinge area where is expressed as Eqn. 3.3. These shear cracks have an average shear crack space $S_{m\theta}$ and keep stable. Therefore shear crack length is constant in this paper.

3.1.4. Estimation results for crack length

Fig. 17 shows the crack length obtained from experimental results and analytical results for the specimens of the load test series 2011. Estimated the sum of flexural-shear crack length is approximate the experimental results, but the estimated crack length divided in flexural zone part and shear zone part are not approximate the experimental results for each specimen. Estimated shear crack lengths of specimen F-60 and F-90 are approximate the experimental results, but that of specimen FS-90 is underestimated the experimental result.

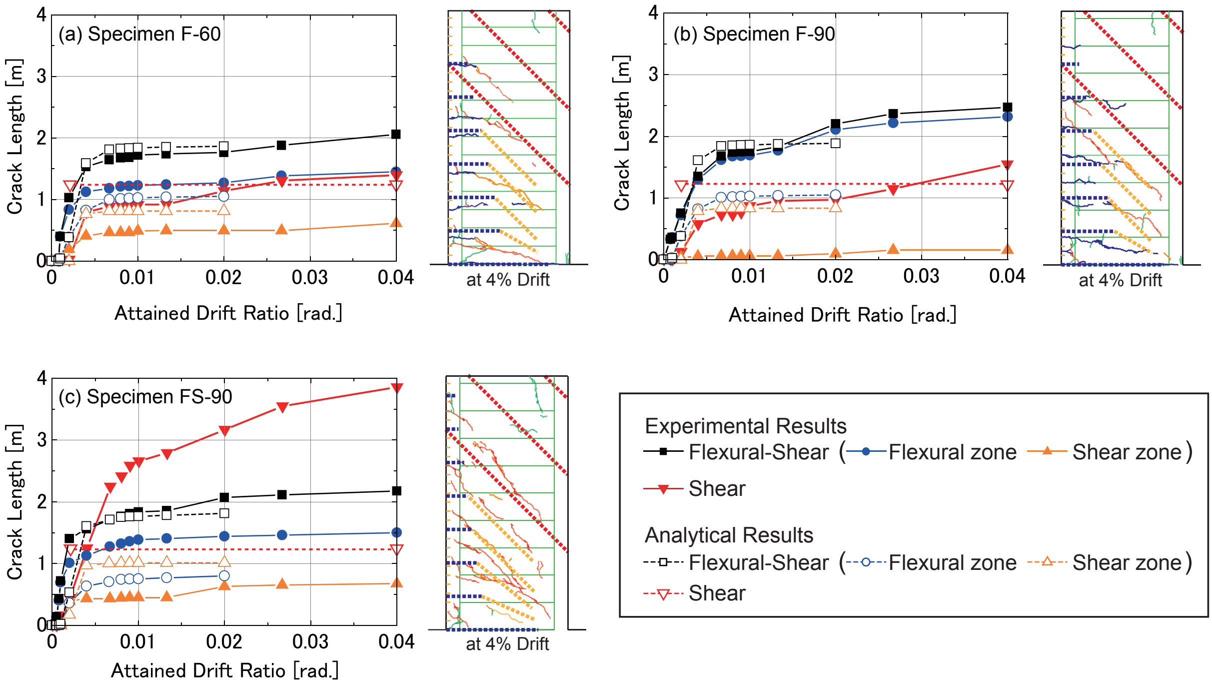


Figure 17. Comparison between the estimated crack length and experimental result

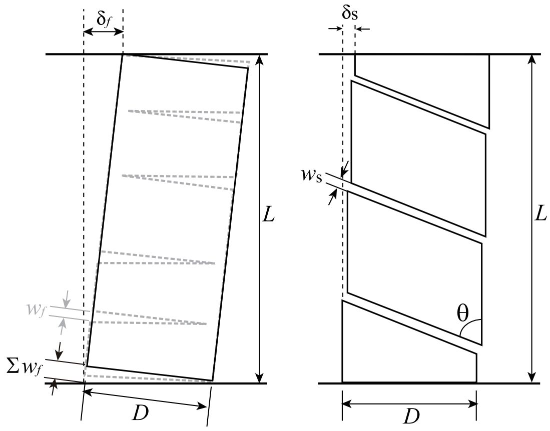
3.2. Crack width estimation model

3.2.1. Geometrical model of crack width and drift

In this paper, geometrical macro model of relation between crack width and drift ratio shown in Fig. 18 (AIJ, 2004) is applied to estimating the residual crack width after excitation. The relation between crack width and drift ratio is expressed as

$$R = R_f + R_s = \frac{\sum w_f}{D - x_n} + \frac{2\sum w_s \cdot \cos\theta}{L} \tag{3.5}$$

where, R_f: current flexural drift ratio, R_s: current shear drift ratio, w_f: flexural crack width, w_s: shear crack width, D: depth, x_n: distance from extreme compression fiber to neutral axis, and L: clear span, respectively. Eqn. 3.5 considers the experimental result of shear crack width and shear drift shown in Fig. 19, which is proposed by Sugi, et al. (Sugi, et al., 2007).



(a) Geometrical model of relation between Flexual crack width - Drift

(b) Geometrical model of relation between Shear crack width - Drift

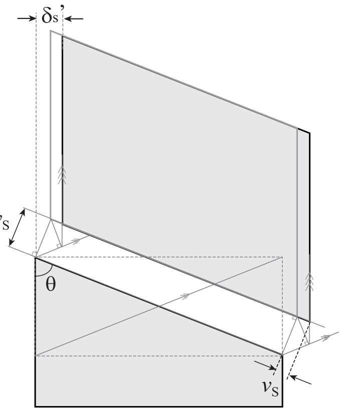


Figure 18. Geometrical model of crack width and drift **Figure 19.** Shear crack width and drift

3.2.2. Estimation results for crack width

Estimation results of the maximum crack width for the specimens of the load test series 2008, which is selected from larger one of the maximum flexural crack width and the maximum shear crack width, are shown in Fig. 20 to 21. The estimated crack width of specimen F-1 can approximately simulate the experimental result. On the contrary, that of specimen S-1 can approximately simulate the experimental result only at the unloaded drift, and it overestimates at the peak drift and underestimates at the zero-residual drift. It implies that the geometrical model shown in Fig. 18 matches up with the unloaded drift condition.

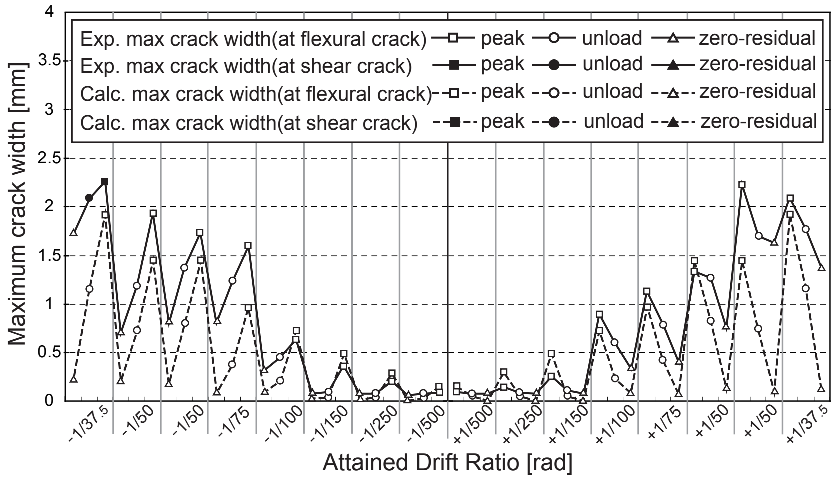


Figure 20. Crack width estimation of specimen F-1

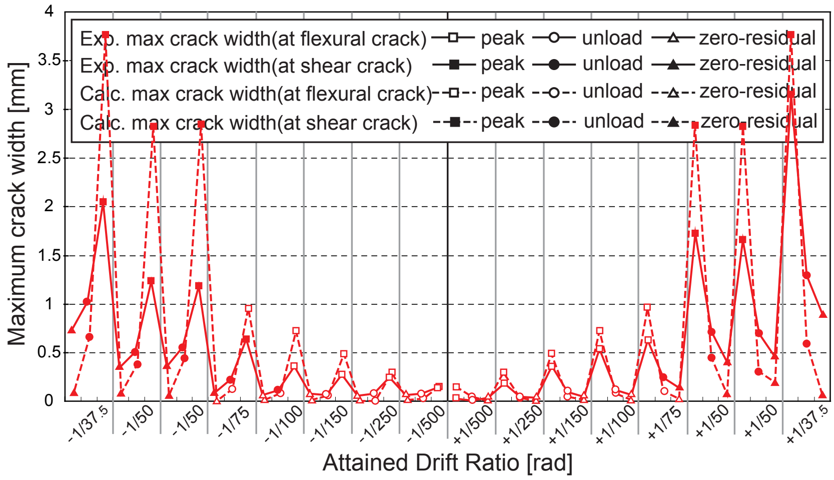


Figure 21. Crack width estimation of specimen S-1

3.3. Probabilistic model for relationship between crack width and length

A probabilistic model between crack widths and lengths is introduced in this study to handle the macro-model such as a geometrical damage estimation model. Crack length distribution to crack width is represented as log-normal distribution based on the previous research (Takimoto, et al., 2004). As concern with the standard deviation, the experimental results are shown in Fig 22. Using the crack widths estimated by the geometrical model and the standard deviation σ obtained from the experimental tests, an example of crack length distribution histograms for the specimens of the load test series 2008 are calculated in Fig. 23 for want of space.

The calculated crack length distribution histograms of specimen F-1 approximately simulate the experimental results. But the trends for underestimating the crack length of a smaller crack width at the peak drift stage are shown in Fig. 23 (a). The calculated crack length distribution histograms of specimen S-1 shows that the trends for underestimating the crack length of a smaller crack width at the peak drift stage and overestimating the crack length of a smaller crack width at the unloaded and zero-residual drift stage as shown in Fig. 23 (b).

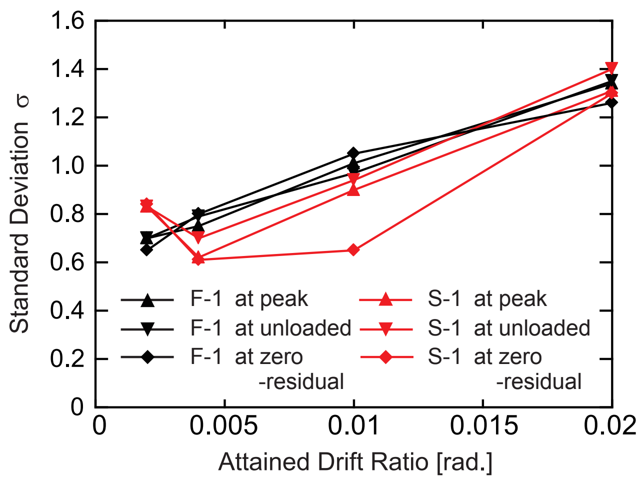
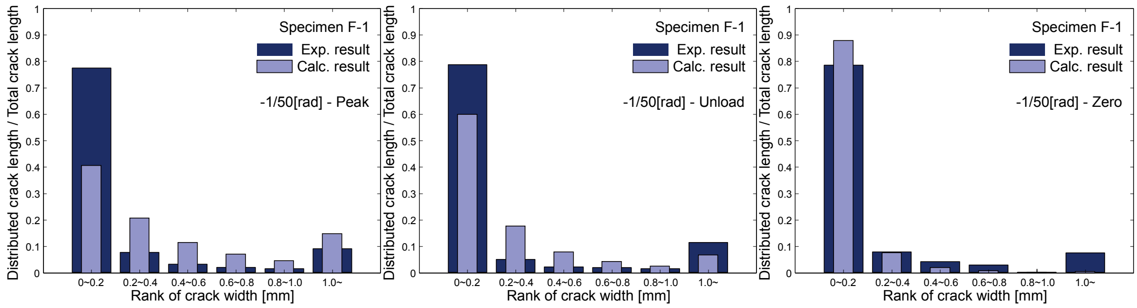
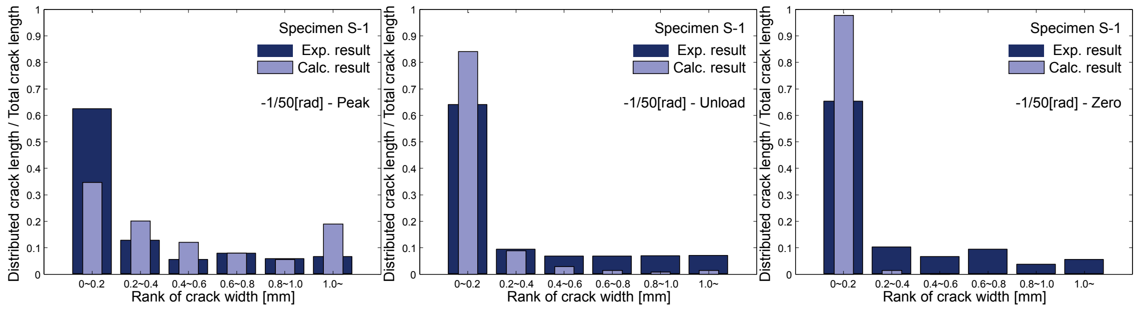


Figure 22. Standard deviation of crack length distribution to crack width



(a) Specimen F-1 at the attained drift ratio = 0.02 rad.



(b) Specimen S-1 at the attained drift ratio = 0.02 rad.

Figure 23. Crack length distribution to crack width

4. CONCLUSION

The proposed models show that the estimated flexure and flexural-shear crack widths and length approximate the measured crack widths and length, but the estimated shear crack widths and length disagree with test results. To revise the model, shear crack and drift mechanism in seismic excitation needs to be more investigated.

ACKNOWLEDGEMENT
The financial support of Japan Society for the Promotion of Science Grant-in-Aid for Young Scientists (Category (B), Grant No. 23760517 and 19760380, Principal Investigator: Noriyuki Takahashi) are gratefully appreciated.

REFERENCES

Architectural Institute of Japan (2004), *Guidelines for Performance Evaluation of Earthquake Resistant Reinforced Concrete Buildings (Draft)*. (in Japanese)

CEB-FIP (1978), *Model Code for Concrete Structures*

Kent D. C. and Park R. (1971), "Flexural members with confined concrete", *Journal of Structural Division, ASCE*, **97**: **ST7**, 1969-1990.

Morita, S. (1969), "Allowable stress design of reinforcing bar due to the restricted concrete crack width", *Annual Report of Cement Technology*, **Vol. 23**, 552-556. (in Japanese)

Okamura, H., and Maekawa, K. (1991). *Nonlinear analysis and constitutive models of reinforced concrete*, Gihodo Publishers. (in Japanese)

Sugi, T., Ishimori, A., Tajima, K., and Shirai, N. (2007), "Damage evaluation of RC beam members based on accurate measurement of displacement and crack widths with scanner (part 2) proposal of rational suggestion of quantitative evaluation model for evaluating of shear crack width – shear deformation relationship", *Summaries of Technical Papers of Annual Meeting of Architectural Institute of Japan,* **C-2**, 373-374. (in Japanese)

Takimoto, K. et al. (2004), "Study on evaluation of damage of RC beams using crack data", *Journal of the Japan Society of Civil Engineers*, **760/V-63**, 135-145. (in Japanese)