13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 149

SIMPLIFIED NONLINEAR ANALYSIS PROCEDURE FOR ASYMMETRIC BUILDINGS

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SUMMARY

A simplified nonlinear analysis procedure to predict earthquake responses of multi-story asymmetric buildings is presented and some examples are shown in this paper. In this procedure, their responses are predicted through a nonlinear static analysis of MDOF model and estimation of seismic demand of equivalent SDOF model, considering the change in the first mode shape at each nonlinear stage and the effect of the first and second mode contribution. The results show that the responses of multi-story asymmetric buildings can be satisfactorily predicted by the proposed procedure.

INTRODUCTION

The estimation of nonlinear response of buildings subjected to a strong ground motion is a key issue for the rational seismic design of new buildings and the seismic evaluation of existing buildings (ATC-40 [1], FEMA 356 [2], Otani [3]). For this purpose, the nonlinear time-history analysis of Multi-Degree-Of-Freedom (MDOF) model might be one solution, but it is often complicated whereas the results are not necessarily more reliable due to uncertainties involved in input data. To overcome such shortcomings, several researchers have developed simplified nonlinear analysis procedures using equivalent Single-Degree-Of-Freedom (SDOF) model (Saiidi and Sozen [4], Fajfar and Fishinger [5], Fajfar [6], and Kuramoto et al. [7]). This approach consists of a nonlinear static (pushover) analysis of MDOF model and a nonlinear dynamic analysis of the equivalent Single-Degree-Of-Freedom (SDOF) model, and it would be a promising candidate as long as buildings oscillate predominantly in the first mode. Although these procedures have been more often applied to planer frame analyses, only limited investigations concerning the extension of the simplified procedure to asymmetric buildings have been made (Moghadam and Tso [8], Fajfar et al. [9]).

In this paper, a simplified nonlinear analysis procedure to predict earthquake responses of low-rise, multi-story asymmetric buildings is presented and some examples are shown. The proposed procedure has

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three advantages over the traditional procedure. Firstly, it does not need the pushover analysis of complicated three-dimensional multi-story building models, but it needs pushover analyses of planar frames and a simplified equivalent single-story model proposed in this paper. Secondly, the change in the first mode shape at each nonlinear stage is considered to determine the equivalent SDOF model properties. Thirdly, it provides a better prediction for the drift demand in each frame from two different pushover analyses considering the effect of the first and second mode contributions.

In the examples, the procedure is applied to 4-story asymmetric shear-building models and the results are compared with the nonlinear dynamic analysis results. Since the simplified nonlinear analysis procedure for single-story asymmetric buildings in the previous study (Fujii et al. [10]) is applicable only to torsionally stiff (TS) buildings (Fajfar et al. [9], Fujii et al. [10]), the discussion in this paper is also limited to TS buildings.

EQUATIONS OF MOTIONS OF THE SIMPLIFIED MODELS

Model Assumptions

The building model considered in this study is an idealized *N*-story asymmetric shear-building model as shown Figure 1. In this paper, following assumptions are made for the model.

1) All floors have the same plan geometry and the same locations of frames.
2) The centers of mass of all floor diaphragms lie on the same vertical axis.
3) All floor diaphragms have the same radius of gyration about the vertical axis through center of mass r as expressed in Eq. (1).

$$r = \sqrt{I_1/m_1} = \sqrt{I_2/m_2} = \ldots = \sqrt{I_N/m_N} \tag{1}$$

where m_i, I_i are mass and moment of inertia of i-th floor, respectively.

4) All frames are oriented in the X or Y-direction and the framing plan is symmetric about the X-axis and properties of the two symmetrically located frames are identical.
5) Each frame consists of column elements and wall elements. The axial deformation of those vertical elements is negligibly small.
6) The ratio of story stiffness and strength in X- and Y-directions is the same for all stories as expressed

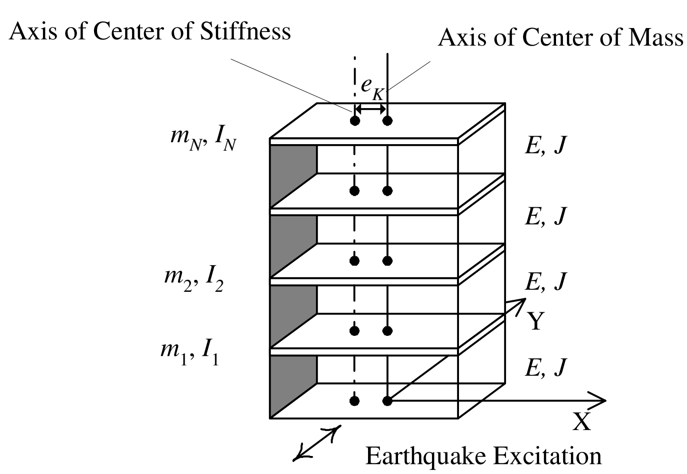


Figure 1 *N*-Story Asymmetric Shear-Building Model

in Eq. (2).

$$\frac{\sum_l K_{Y1l}}{\sum_l K_{X1l}}=\frac{\sum_l K_{Y2l}}{\sum_l K_{X2l}}=\cdots=\frac{\sum_l K_{YNl}}{\sum_l K_{XNl}}=const.,\qquad \frac{\sum_l V_{yY1l}}{\sum_l V_{yX1l}}=\frac{\sum_l V_{yY2l}}{\sum_l V_{yX2l}}=\cdots=\frac{\sum_l V_{yYNl}}{\sum_l V_{yXNl}}=const. \tag{2}$$

where K_{Xil}, and K_{Yil} are the stiffness of the l-th frame in i-th story oriented in X- and Y-direction, respectively, and V_{yXil}, and V_{yYil} are the yield strength of the l-th frame in i-th story oriented in X- and Y-direction, respectively.

7) All frames have the same vertical distribution of stiffness and strength for the story as expressed in Eq. (3).

$$\frac{K_{Xi1}}{K_{X11}}=\frac{K_{Xi2}}{K_{X12}}=\cdots=\frac{K_{Yi1}}{K_{Y11}}=\frac{K_{Yi2}}{K_{Y12}}=\cdots=const.,\qquad \frac{V_{yXi1}}{V_{yX11}}=\frac{V_{yXi2}}{V_{yX12}}=\cdots=\frac{V_{yYi1}}{V_{yY11}}=\frac{V_{yYi2}}{V_{yY12}}=\cdots=const. \tag{3}$$

From these assumptions, the centers of stiffness of all stories lie on the same vertical axis: the eccentricity ratio E (= e_K / r, e_K : elastic eccentricity) and the radius ratio of gyrations of story stiffness J (= j / r, j : radius of gyration of story stiffness) are the same in all stories.

In this paper, the earthquake excitation is considered unidirectional in Y-direction. Therefore 2N degrees of freedom (DOFs) are considered for the multi-story model.

Equivalent Single-Story Model

Natural vibration modes and equivalent modal mass ratio of multi-story asymmetric building model

The equation of motion of undamped free vibration of N-story asymmetric building model shown in Figure 1 can be written as Eq. (4).

$$\begin{bmatrix}[m] & [0]\\ [0] & [I]\end{bmatrix}\begin{Bmatrix}\{\ddot{y}\}\\ \{\ddot{\theta}\}\end{Bmatrix}+\begin{bmatrix}[K_Y] & -e_K\cdot[K_Y]\\ -e_K\cdot[K_Y] & j^2\cdot[K_Y]\end{bmatrix}\begin{Bmatrix}\{y\}\\ \{\theta\}\end{Bmatrix}=\begin{Bmatrix}\{0\}\\ \{0\}\end{Bmatrix} \tag{4}$$

Where $[M]=\begin{bmatrix}[m] & [0]\\ [0] & [I]\end{bmatrix}$: mass matrix of N-story asymmetric building model, $\{d\}=\begin{Bmatrix}\{y\}\\ \{\theta\}\end{Bmatrix}$: vector representing displacement at the center of mass of floor diaphragms and $[K]=\begin{bmatrix}[K_Y] & -e_K\cdot[K_Y]\\ -e_K\cdot[K_Y] & j^2\cdot[K_Y]\end{bmatrix}$ is the elastic stiffness matrix of N-story asymmetric building model. By substituting Eq. (5) into Eq. (4), Eq. (6) is obtained.

$$\{z\}=r\cdot\{\theta\} \tag{5}$$

$$\begin{bmatrix}[m] & [0]\\ [0] & [m]\end{bmatrix}\begin{Bmatrix}\{\ddot{y}\}\\ \{\ddot{z}\}\end{Bmatrix}+\begin{bmatrix}[K_Y] & -E\cdot[K_Y]\\ -E\cdot[K_Y] & J^2\cdot[K_Y]\end{bmatrix}\begin{Bmatrix}\{y\}\\ \{z\}\end{Bmatrix}=\begin{Bmatrix}\{0\}\\ \{0\}\end{Bmatrix} \tag{6}$$

From Eq. (6), the k-th natural frequency ω_k and mode $\{\phi_k\}$ of the N-story asymmetric building models can be obtained from Eqs. (7) to (10) (Shiga [11], Chopra [12]).

$$\omega_k=\omega_{Ti}\cdot\omega_{Sj} \tag{7}$$

$$\{\phi_k\} = \left\{\phi_{TYi}\cdot\{\phi_{Sj}\}^T \quad \phi_{TZi}\cdot\{\phi_{Sj}\}^T\right\}^T \tag{8}$$

$$\left(-\omega_{Ti}^{\ 2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}1 & -E\\ -E & J^2\end{bmatrix}\right)\begin{Bmatrix}\phi_{TYi}\\ \phi_{TZi}\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix} \tag{9}$$

$$\left(-\omega_{Sj}^{\ 2}[m]+[K_Y]\right)\{\phi_{Sj}\}=\{0\} \tag{10}$$

where ω_{Ti} and $\{\phi_{Ti}\}(=\{\phi_{TYi}, \phi_{TZi}\}^T)$ are i-th natural frequency ratio and mode of the associated single-story model whose eccentricity ratio and radius ratio of gyration are E and J, respectively, and ω_{Sj} and $\{\phi_{Sj}\}$ are the j-th natural frequency and mode of the associated N-story symmetric building model. In this paper, this associated single-story model is referred to as "equivalent single-story model". In this paper, $\{\phi_{Ti}\}$ and $\{\phi_{Sj}\}$ are normalized so that ϕ_{TYi} equal 1.0 and the component of the top of $\{\phi_{Sj}\}$ equal 1.0.
The first modal participation factor Γ_1 and the first equivalent modal mass ratio of N-story asymmetric building model m_1^* can be written by Eq. (11).

$$\Gamma_1=\frac{\{\phi_1\}^T[M]\{\alpha\}}{\{\phi_1\}^T[M]\{\phi_1\}}, m_1^*=\frac{M_1^*}{\sum_{i=1}^{N}m_i}=\frac{\Gamma_1\left(\{\phi_1\}^T[M]\{\phi_1\}\right)}{\sum_{i=1}^{N}m_i} \tag{11}$$

Where $\{\alpha\} = \{\{1\}^T, \{0\}^T\}^T$ is the vector defining the direction of ground motion. From Eqs. (7) to (10), Eq. (11) can be rewritten as Eq. (12).

$$\Gamma_1=\Gamma_{T1}\cdot\Gamma_{S1}, m_1^*=m_{T1}^*\cdot m_{S1}^* \tag{12}$$

where Γ_{T1} and Γ_{S1} are the first modal participation factor of the equivalent single-story model and the associated N-story symmetric shear-building model expressed in Eq. (13), respectively, and m_{T1}^* and m_{S1}^* are the first modal mass ratio of the equivalent single-story model and the associated N-story symmetric building model expressed in Eq. (14), respectively.

$$\Gamma_{T1}=\frac{\phi_{TY1}}{\phi_{TY1}^{\ 2}+\phi_{TZ1}^{\ 2}}=\frac{\phi_{TY1}}{\phi_{TY1}^{\ 2}+(r\cdot\phi_{T\Theta1})^2}, \Gamma_{S1}=\frac{\{\phi_{S1}\}^T[m]\{1\}}{\{\phi_{S1}\}^T[m]\{\phi_{S1}\}} \tag{13}$$

$$m_{T1}^*=\Gamma_{T1}\cdot\phi_{TY1}=\frac{\phi_{TY1}^{\ 2}}{\phi_{TY1}^{\ 2}+(r\cdot\phi_{T\Theta1})^2}, m_{S1}^*=\frac{1}{\sum_{i=1}^{N}m_i}\cdot\Gamma_{S1}\{\phi_{S1}\}^T[m]\{1\}=\frac{1}{\sum_{i=1}^{N}m_i}\cdot\frac{\left(\{\phi_{S1}\}^T[m]\{1\}\right)^2}{\{\phi_{S1}\}^T[m]\{\phi_{S1}\}} \tag{14}$$

Note that the first equivalent modal mass ratio of N-story asymmetric shear-building model m_1^* is the product of that of equivalent single-story model m_{T1}^* and that of the associated N-story symmetric shear-building model m_{S1}^*. Figure 2 shows the contour line of m_{T1}^* on E-J plane for the equivalent single-story model, and Figure 3 shows the relationship of m_{S1}^* and N for the associated N-story symmetric shear-building model having the same floor mass and inverted-triangular first mode shape ($N \leq 7$). Figure 2 shows that m_{T1}^* is larger than 0.5 for torsionally stiff (TS) building ($J > 1$) and smaller than 0.5 for torsionally flexible (TF) building ($J < 1$), as discussed in the previous study (Fujii et al. [10]): for Model-A and Model-B studied in ANALYSIS EXAMPLES, m_{T1}^* equals 0.829 and 0.743, respectively. And Figure 3 shows that for the associated N-story symmetric shear-building model, m_{S1}^* is larger than 0.8: for 4-story symmetric shear-building model, m_{S1}^* equals 0.833. They imply that the multi-story TS buildings have relatively larger m_1^* value. As is shown in the previous study (Fujii et al. [10]), m_1^* can be a good index for the first modal contribution to overall response. Therefore response of the multi-story TS buildings may be predominantly governed by the first mode.

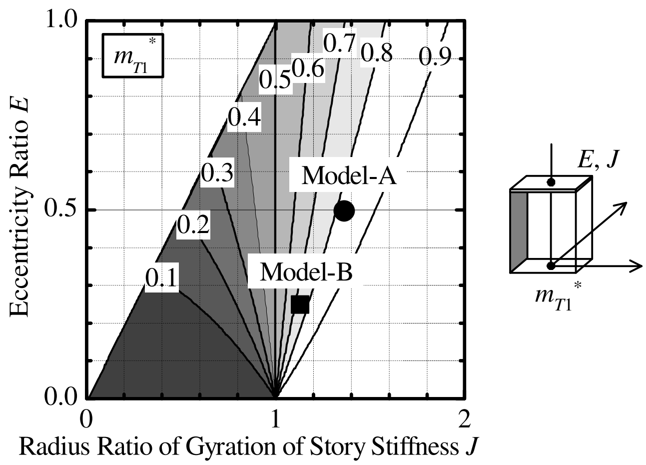


Figure 2 Contour line of m_{T1}^* in E-J Plane

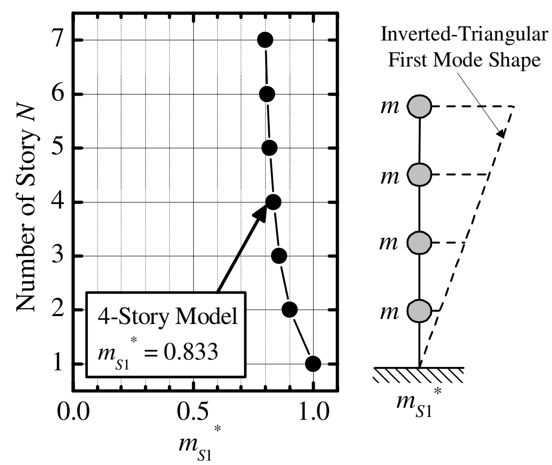


Figure 3 Relationship of m_{S1}^* and N

Equation of motion of equivalent single-story model

The equation of motion of N-story asymmetric building model subjected to unidirectional ground motion can be written as Eq. (15).

$$\begin{bmatrix} [m] & [0] \\ [0] & [I] \end{bmatrix} \begin{Bmatrix} \{\ddot{y}\} \\ \{\ddot{\theta}\} \end{Bmatrix} + \begin{bmatrix} [C_{YY}] & [C_{Y\theta}] \\ [C_{\theta Y}] & [C_{\theta\theta}] \end{bmatrix} \begin{Bmatrix} \{\dot{y}\} \\ \{\dot{\theta}\} \end{Bmatrix} + \begin{Bmatrix} \{R_Y\} \\ \{M_Z\} \end{Bmatrix} = - \begin{bmatrix} [m] & [0] \\ [0] & [I] \end{bmatrix} \{\alpha\} \cdot a_g \tag{15}$$

Where $[C] = \begin{bmatrix} [C_{YY}] & [C_{Y\theta}] \\ [C_{\theta Y}] & [C_{\theta\theta}] \end{bmatrix}$: damping matrix of N-story asymmetric building model, $\{R\} = \begin{Bmatrix} \{R_Y\} \\ \{M_Z\} \end{Bmatrix}$: vector representing restoring force at the center of mass of floor diaphragms and a_g is the ground acceleration. The j-th mode of the associated N-story symmetric shear-building model $\Gamma_{Sj}\,\{\phi_{Sj}\}$ is assumed constant even if the building oscillates beyond elastic range. The displacement vector $\{d\}$ is assumed in the form of Eq. (16).

$$\{d\} = \begin{Bmatrix} \{y\} \\ \{\theta\} \end{Bmatrix} = \left\{ \sum_{j=1}^{N} \Gamma_{Sj} \{\phi_{Sj}\}^T \cdot Y_j^* \quad \sum_{j=1}^{N} \Gamma_{Sj} \{\phi_{Sj}\}^T \cdot \Theta_j^* \right\}^T \tag{16}$$

Where Y_j^* and Θ_j^* are the displacement at the center of mass and the rotation corresponding to the j-th mode of the associated N-story symmetric shear-building model. Eq. (16) is rewritten as Eq. (17), assuming the predominant oscillation of the first mode of the associated N-story symmetric shear-building model.

$$\{d\} = \left\{ \{y\}^T \quad \{\theta\}^T \right\}^T = \left\{ \Gamma_{S1} \{\phi_{S1}\}^T \cdot Y_1^* \quad \Gamma_{S1} \{\phi_{S1}\}^T \cdot \Theta_1^* \right\}^T \tag{17}$$

Substituting Eq. (17), Eq. (15) can be rewritten as Eq. (18).

$$\begin{cases} [m](\Gamma_{S1}\{\phi_{S1}\}) \cdot \ddot{Y}_1^* + [C_{YY}](\Gamma_{S1}\{\phi_{S1}\}) \cdot \dot{Y}_1^* + [C_{Y\theta}](\Gamma_{S1}\{\phi_{S1}\}) \cdot \dot{\Theta}_1^* + \{R_Y\} = -[m]\{1\} \cdot a_g \\ [I](\Gamma_{S1}\{\phi_{S1}\}) \cdot \ddot{\Theta}_1^* + [C_{\theta Y}](\Gamma_{S1}\{\phi_{S1}\}) \cdot \dot{Y}_1^* + [C_{\theta\theta}](\Gamma_{S1}\{\phi_{S1}\}) \cdot \dot{\Theta}_1^* + \{M_Z\} = \{0\} \end{cases} \tag{18}$$

By multiplying $\Gamma_{S1}\{\phi_{S1}\}^T$ from left side of Eq. (18), the equation of motion of equivalent single-story model is obtained as Eqs. (19) through (22).

$$\begin{bmatrix} M_{T1}^* & 0 \\ 0 & I_{T1}^* \end{bmatrix} \begin{Bmatrix} \ddot{Y}_1^* \\ \ddot{\Theta}_1^* \end{Bmatrix} + \begin{bmatrix} C_{YYT1}^* & C_{Y\theta T1}^* \\ C_{\theta YT1}^* & C_{\theta\theta T1}^* \end{bmatrix} \begin{Bmatrix} \dot{Y}_1^* \\ \dot{\Theta}_1^* \end{Bmatrix} + \begin{Bmatrix} V_Y^* \\ T_Z^* \end{Bmatrix} = -\begin{bmatrix} M_{T1}^* & 0 \\ 0 & I_{T1}^* \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \cdot a_g \tag{19}$$

$$M_{T1}^* = \Gamma_{S1}^{\ 2}\left(\{\phi_{S1}\}^T[m]\{\phi_{S1}\}\right), I_{T1}^* = \Gamma_{S1}^{\ 2}\left(\{\phi_{S1}\}^T[I]\{\phi_{S1}\}\right) \tag{20}$$

$$\left[C_{T1}^*\right] = \begin{bmatrix} C_{YYT1}^* & C_{Y\theta T1}^* \\ C_{\theta YT1}^* & C_{\theta\theta T1}^* \end{bmatrix} = \begin{bmatrix} \Gamma_{S1}^{\ 2}\left(\{\phi_{S1}\}^T[C_{YY}]\{\phi_{S1}\}\right) & \Gamma_{S1}^{\ 2}\left(\{\phi_{S1}\}^T[C_{Y\theta}]\{\phi_{S1}\}\right) \\ \Gamma_{S1}^{\ 2}\left(\{\phi_{S1}\}^T[C_{\theta Y}]\{\phi_{S1}\}\right) & \Gamma_{S1}^{\ 2}\left(\{\phi_{S1}\}^T[C_{\theta\theta}]\{\phi_{S1}\}\right) \end{bmatrix} \tag{21}$$

$$\left\{D_{T1}^*\right\} = \begin{Bmatrix} Y_1^* \\ \Theta_1^* \end{Bmatrix} = \begin{Bmatrix} \Gamma_{S1}\{\phi_{S1}\}^T[m]\{y\}/M_{T1}^* \\ \Gamma_{S1}\{\phi_{S1}\}^T[I]\{\theta\}/I_{T1}^* \end{Bmatrix}, \left\{R_{T1}^*\right\} = \begin{Bmatrix} V_Y^* \\ T_Z^* \end{Bmatrix} = \begin{Bmatrix} \Gamma_{S1}\{\phi_{S1}\}^T\{R_Y\} \\ \Gamma_{S1}\{\phi_{S1}\}^T\{M_Z\} \end{Bmatrix} \tag{22}$$

where M_{T1}^* and I_{T1}^* are the equivalent mass and equivalent moment of inertia of equivalent single-story model, respectively, and $[C_{T1}^*]$ is the damping matrix of equivalent single-story model, and $\{D_{T1}^*\}$ and $\{R_{T1}^*\}$ are the displacement and restoring force vector of the equivalent single-story model, respectively.

Equivalent SDOF Model

In the equation of motion of equivalent single-story model (Eqs. (19) through (22)), the displacement $\{D_{T1}^*\}$ and restoring force vector $\{R_{T1}^*\}$ are assumed in the form of Eq. (23) even if the building responds beyond the elastic range.

$$\left\{D_{T1}^*\right\} = \begin{Bmatrix} Y_1^* \\ \Theta_1^* \end{Bmatrix} = \Gamma_{T1}\{\phi_{T1}\} \cdot D_1^* + \Gamma_{T2}\{\phi_{T2}\} \cdot D_2^*, \left\{R_{T1}^*\right\} = \begin{Bmatrix} V_Y^* \\ T_Z^* \end{Bmatrix} = \begin{bmatrix} M_{T1}^* & 0 \\ 0 & I_{T1}^* \end{bmatrix} \left(\Gamma_{T1}\{\phi_{T1}\} \cdot A_1^* + \Gamma_{T2}\{\phi_{T2}\} \cdot A_2^*\right) \tag{23}$$

where D_i^* and A_i^* are the i-th equivalent modal displacement and equivalent modal acceleration, respectively. Note that mode shape of the equivalent single-story model $\{\phi_{Ti}\}$ *varies* depending on the stiffness degradation. In this study, $\{\phi_{Ti}\}$ is determined from the secant stiffness defined at the maximum deformation previously experienced in the calculation.

Eq. (23) can be rewritten as Eq. (24), by assuming the predominant oscillation of the first mode and neglecting the second mode of the equivalent single-story model.

$$\left\{D_{T1}^*\right\} = \begin{Bmatrix} Y_1^* \\ \Theta_1^* \end{Bmatrix} = \Gamma_{T1}\{\phi_{T1}\} \cdot D_1^*, \left\{R_{T1}^*\right\} = \begin{Bmatrix} V_Y^* \\ T_Z^* \end{Bmatrix} = \begin{bmatrix} M_{T1}^* & 0 \\ 0 & I_{T1}^* \end{bmatrix} \left(\Gamma_{T1}\{\phi_{T1}\} \cdot A_1^*\right) \tag{24}$$

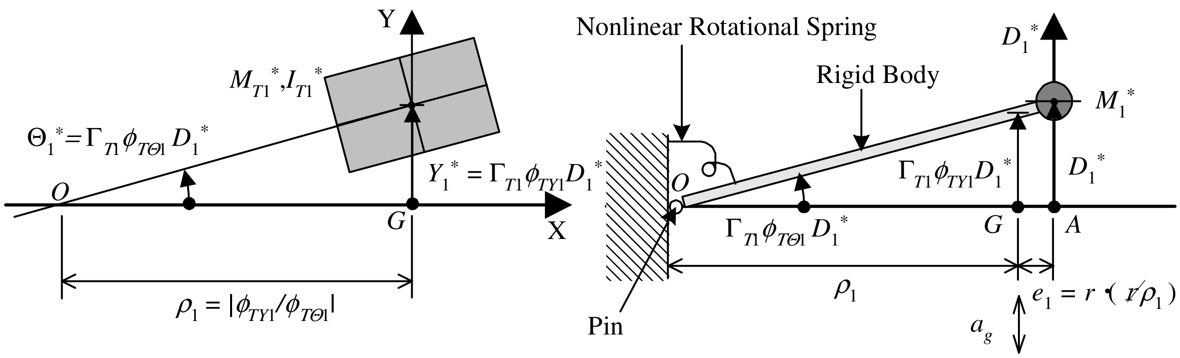
By substituting Eq. (24) into Eq. (19) and by multiplying $\Gamma_{T1}\{\phi_{T1}\}^T$ from the left side, the equation of motion of equivalent SDOF model is obtained as Eq. (25).

$$\ddot{D}_1^* + \frac{C_1^*}{M_1^*} \cdot \dot{D}_1^* + A_1^* = -a_g \tag{25}$$

where M_1^* is the first equivalent modal mass and C_1^* is the first equivalent modal damping coefficient. M_1^* and C_1^* are expressed as Eq. (26).

$$M_1^* = \Gamma_{T1}\{\phi_{TY1} \quad \phi_{T\Theta 1}\}\begin{bmatrix} M_{T1}^* & 0 \\ 0 & I_{T1}^* \end{bmatrix}\begin{Bmatrix} 1 \\ 0 \end{Bmatrix}, C_1^* = \Gamma_{T1}^2\left(\{\phi_{TY1} \quad \phi_{T\Theta 1}\}\begin{bmatrix} C_{YYT1}^* & C_{Y\theta T1}^* \\ C_{\theta YT1}^* & C_{\theta\theta T1}^* \end{bmatrix}\begin{Bmatrix} \phi_{TY1} \\ \phi_{T\Theta 1} \end{Bmatrix}\right) \tag{26}$$

Figures 4(a) and 4(b) show the first mode shape of the equivalent single-story model and corresponding equivalent SDOF model, respectively.



(a) Equivalent Single-Story Model

(b) Equivalent SDOF Model

Figure 4 First Mode of Equivalent Single-Story Model and Equivalent SDOF Model

In Figure 4(b), the equivalent SDOF model consists of a concentrated mass M_1^* located at A, rigid body OA, pin connected to the base at O, and nonlinear rotational spring. Denoting that the distance from the center of mass of equivalent single-story model G to A is e_1, Eq. (27) is obtained from Figure 4:

$$\frac{\rho_1}{e_1 + \rho_1} = \Gamma_{T1} \cdot \phi_{TY1} = \frac{\phi_{TY1}^{\ \ 2}}{\phi_{TY1}^{\ \ 2} + (r \cdot \phi_{T\Theta 1})^2} \tag{27}$$

where ρ_1 is the distance from G to O expressed as Eq. (28).

$$\rho_1 = \left|\frac{\phi_{TY1}}{\phi_{T\Theta 1}}\right| \tag{28}$$

From Eqs. (27) and (28), Eq. (29) is obtained.

$$e_1 = r \cdot \left(\frac{r}{\rho_1}\right) \tag{29}$$

Eq. (29) indicates that distance from the center of mass of equivalent single-story model G to the location of mass A is inversely proportional to the distance from G and the pin O. Therefore, if the first mode response is purely translational in Y-direction $(\rho_1 = \infty)$, e_1 equals 0 and the location of the mass A coincides with G.

Eq. (29) can also be obtained from the following manner. In Figure 4(a), the moment about point O, M_{Oa}, can be expressed by Eq. (30). And in Figure 2(b), the moment about point O, M_{Ob}, can be expressed by Eq. (31). Eq. (29) is obtained by equating M_{oa} and M_{ob} in Eqs. (30) and (31).

$$M_{oa} = M_{T1}^{*} \cdot \Gamma_{T1}\phi_{TY1} \cdot A_1^{*} \cdot \rho_1 + I_{T1}^{*} \cdot \Gamma_{T1}\phi_{T\Theta 1} \cdot A_1^{*} \quad (30)$$

$$M_{ob} = M_{T1}^{*} \cdot A_1^{*} \cdot (\rho_1 + e_1) \quad (31)$$

DESCRIPTIONS OF PROPOSED SIMPLIFIED PROCEDURE

In this chapter, a simplified nonlinear analysis procedure for multi-story asymmetric building is proposed. The outline of the proposed procedure is described as follows.

STEP 1: Pushover Analysis of planar frame
STEP 2: Pushover Analysis of equivalent single-story model
STEP 3: Estimation of the seismic demand of equivalent SDOF model
STEP 4: Estimation of drift demand in each frame of equivalent single-story model
STEP 5: Estimation of drift demand in each story of planar frames

Figure 5 shows the outline of the proposed procedure.

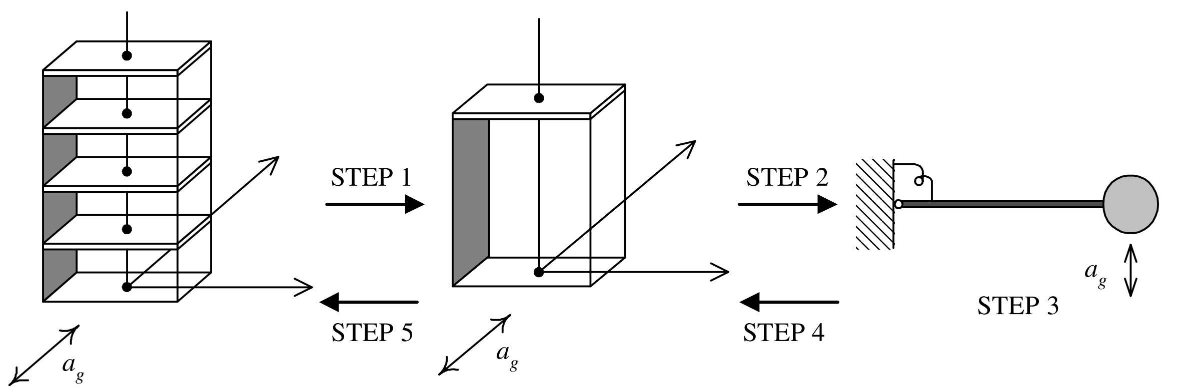


Figure 5 Outline of the proposed procedure

STEP 1: Pushover Analysis of Planar Frame

Pushover analysis of each planar frame is carried out to obtain its non-linear force-displacement relationship, assuming *invariant* vertical distribution of displacement $\Gamma_{S1}\{\phi_{S1}\}$ for all frames.

The properties of the equivalent single-story model are determined from the results of these pushover analyses. The equivalent mass and equivalent moment of inertia of equivalent single-story model, M_{T1}^{*}, I_{T1}^{*} are determined from Eq. (20), the equivalent restoring force of each frame V_{Xi}^{*} and V_{Yi}^{*} from Eq. (32), and the equivalent displacement of each frame d_{Xi}^{*} and d_{Yi}^{*} from Eq. (33), respectively.

$$V_{Xi}^{*} = \Gamma_{S1}\{\phi_{S1}\}^T\{R_{Xi}\}, V_{Yi}^{*} = \Gamma_{S1}\{\phi_{S1}\}^T\{R_{Yi}\} \quad (32)$$

$$d_{Xi}^{*} = \Gamma_{S1}\{\phi_{S1}\}^T[m]\{d_{Xi}\}/M_{T1}^{*}, d_{Yi}^{*} = \Gamma_{S1}\{\phi_{S1}\}^T[m]\{d_{Yi}\}/M_{T1}^{*} \quad (33)$$

where $\{R_{Xi}\}$,$\{R_{Yi}\}$ and $\{d_{Xi}\}$, $\{d_{Yi}\}$ are restoring force vector and displacement vector of each frame obtained by pushover analysis, respectively.

STEP 2: Pushover Analysis of Equivalent Single-Story Model

Pushover analysis of an equivalent single-story model is carried out to obtain the force-displacement relationship, considering the change in the fundamental mode shape at each nonlinear stage. The numerical procedure of the pushover analysis can be found in APPENDIX.

The property of the equivalent SDOF model is determined from the results of the pushover analysis. Since the deformation shape $\{D_{T1}^*\}$ having the first mode shape is imposed on the equivalent single-story model as described in APPENDIX, the equivalent acceleration A_1^*, equivalent displacement D_1^* of the equivalent SDOF model are determined by the Eqs. (34).

$$A_1^* = \frac{1}{M_{T1}^*}\Gamma_{T1}\{\phi_{TY1} \quad \phi_{T\Theta 1}\}\begin{Bmatrix} V_Y^* \\ T_Z^* \end{Bmatrix} = \left\{1+\left(\frac{\Theta_1^*}{Y_1^*}\right)\cdot\left(\frac{T_Z^*}{V_Y^*}\right)\right\}\cdot\frac{V_Y^*}{M_{T1}^*}, D_1^* = \frac{Y_1^*}{\Gamma_{T1}} = \left\{1+\frac{I_{T1}^*}{M_{T1}^*}\cdot\left(\frac{\Theta_1^*}{Y_1^*}\right)^2\right\}\cdot Y_1^* \qquad (34)$$

The A_1^*-D_1^* relationship, referred to as capacity diagram, are idealized by elasto-plastic bi-linear curve so that the hysteretic dissipation enclosed by the original curve and the bi-linear idealized curve is same.

STEP 3: Estimation of Seismic Demand of Equivalent SDOF Model

The seismic demand of equivalent SDOF model $D_{1\,MAX}^*$ is obtained by the equivalent linearization procedure (Otani [3]) in this study. The equivalent period T_{eq} and equivalent damping ratio h_{eq} of the equivalent SDOF model at each nonlinear stage is calculated by Eq. (35).

$$T_{eq} = 2\pi\sqrt{\frac{D_1^*}{A_1^*}}, h_{eq} = 0.25\left(1-\frac{1}{\sqrt{\mu}}\right)+h_0 = 0.25\left(1-\sqrt{\frac{D_{1Y}^*}{D_1^*}}\right)+h_0 \qquad (35)$$

Where μ is the ductility ratio of equivalent SDOF model, D_{1Y}^* is the yield displacement of the equivalent SDOF model determined from bi-linear curve, and h_0 is the initial damping ratio. In this study, h_0 is assumed 0.03, because in the dynamic time-history analysis of MDOF model the damping is assumed also 3% of critical for the first mode. The response spectral acceleration and displacement are reduced by following factor calculated by Eq. (36).

$$F_h = 1.5/\left(1+10h_{eq}\right) \qquad (36)$$

The damping spectrum of an earthquake excitation is constructed by plotting a SDOF response acceleration $S_A(T_{eq}, h_{eq})$ in the vertical axis and corresponding displacement $S_D(T_{eq}, h_{eq})$ in the horizontal axis. The seismic demand of equivalent SDOF model is determined by comparing the capacity diagram and the demand spectrum. The intersection of the capacity diagram and demand spectrum represents the maximum response of the equivalent SDOF model.

STEP 4: Estimation of Drift Demand in Each Frame of Equivalent Single-Story Model

The drift in each frame of the equivalent single-story model based on the first mode response is determined from the results of STEP 2 and 3. As discussed in the previous study (Fujii et al. [10]), $\{D_{T1}^*\}$ is obtained by substituting $D_{1\,MAX}^*$ obtained in STEP 3 into Eq. (24), and hence the drift demand in each frame based on the first mode response can be found. Then, another pushover analysis is carried out using the sum of modal force distribution determined from Eq. (37) until D^* obtained by Eq. (38) reaches

$D_1{}^*_{MAX}$. The drift demand in each frame of the equivalent single-story model is determined from the envelope of both pushover analyses.

$$\{f\}_+ = \begin{bmatrix} M_{T1}{}^* & 0 \\ 0 & I_{T1}{}^* \end{bmatrix} \left(\Gamma_{T1ie}\{\phi_{T1ie}\} + \Gamma_{T2ie}\{\phi_{T2ie}\} \right) = \begin{bmatrix} M_{T1}{}^* & 0 \\ 0 & I_{T1}{}^* \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \tag{37}$$

$$D^* = \frac{Y_1^*}{\Gamma_{T1ie}} = \left\{ 1 + \left(\frac{I_{T1}{}^*}{M_{T1}{}^*} \cdot \frac{\phi_{T\Theta1ie}}{\phi_{TY1ie}} \right) \cdot \frac{\Theta_1^*}{Y_1^*} \right\} \cdot Y_1^* \tag{38}$$

Where $\Gamma_{T1ie}\{\phi_{T1}\}$ and $\Gamma_{T2ie}\{\phi_{T2}\}$ are the first and second mode shape at $D_1{}^*_{MAX}$, respectively.

STEP 5: Estimation of drift demand in each story of planar frames

The drift demand in each story of frame is determined from the results of STEP 1 and 4. By substituting $d_{Xi}{}^*_{MAX}$ and $d_{Yi}{}^*_{MAX}$ obtained in STEP 4 into Eq. (39), $\{d_{XiMAX}\}$ and $\{d_{YiMAX}\}$ and hence the drift demand in each story of frame can be found.

$$\{d_{XiMAX}\} = \Gamma_{S1}\{\phi_{S1}\} \cdot d_{Xi}{}^*{}_{MAX}, \{d_{YiMAX}\} = \Gamma_{S1}\{\phi_{S1}\} \cdot d_{Yi}{}^*{}_{MAX} \tag{39}$$

ANALYSIS EXAMPLES

Building Data and Ground Motion Data

Building Data

Buildings investigated in this paper are idealized four-story asymmetric shear-building models: they are assumed to be symmetric about the X-axis as shown in Figure 4. Their story height is assumed 3.60 m for all stories and the unit mass is assumed 1.2 x 10^3 kg/m^2. In this study, two structural plans are studied as shown in Figure 6. In each building model, the column element and wall element are placed in frames oriented in X or Y-direction. The yield strengths of the *i*-th story in X and Y-direction, V_i, are determined by Eq. (40): the yield base shear in X and Y-direction are assumed 0.60W(where W is the total weight of the building model).

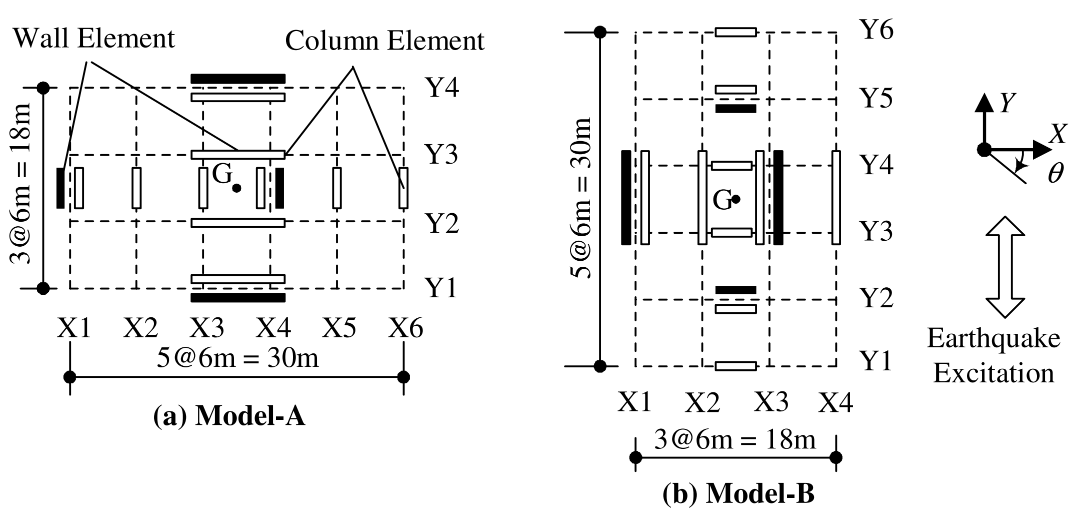
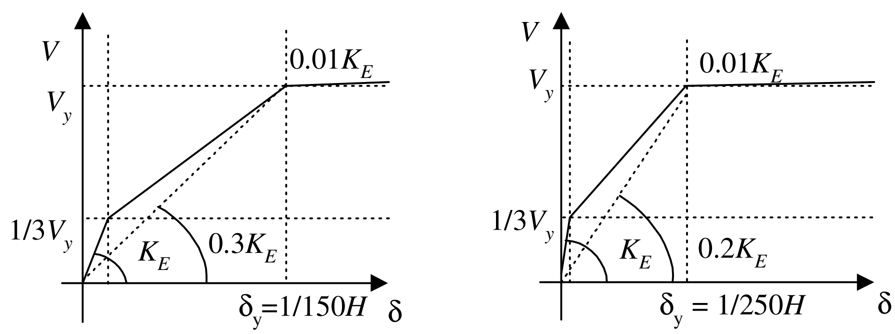


Figure 6 Plan of the Model Buildings

$$V_i = \frac{N+i}{N+1} \cdot 0.6 \cdot \sum_{j=i}^{N} w \qquad (40)$$

Where w is the weight of each floor. The total yield strength of column elements are assumed one-third of V_i, and that of wall element are assumed two-third of V_i. Figure 7 shows the envelope curve of restoring force-drift relationship of each element. The envelopes are assumed symmetric in both positive and negative loading directions. The Takeda hysteretic model (Takeda et al. [13]) is employed for both column and wall elements, assuming that they behave in a ductile manner.

Table 1 shows the yield strengths of column and wall elements, and the model parameters: eccentricity ratio E, radius ratio of gyration of story stiffness J, and eccentricity ratio in accordance with the Japanese Standard of Seismic Design of Buildings Re. Figure 2 and this table show that J is larger than 1.0 in both building models. Figure 8 shows the mode shapes of each building models. As shown in this figure, the first modes of all models are governed by the translational component, while their second mode is governed by the torsional component. Consequently, all models can be classified as torsionally stiff buildings (Fajfar et al. [9], Fujii et al. [10]). Note that the first equivalent modal mass ratios of both



(a) Column Element **(b) Wall Element**

Figure 7 Envelope of Restoring Force-Drift Relationship

Table 1 Yield Strength of Element and Model Parameters

	Yield Strength of Element				E	J	Re
	X-Direction		Y-Direction				
	Column	Wall	Column	Wall			
Model-A	1/18 V_i	1/3 V_i	1/12 V_i	1/3 V_i	0.495	1.365	0.389
Model-B	1/12 V_i		1/18 V_i		0.248	1.129	0.225

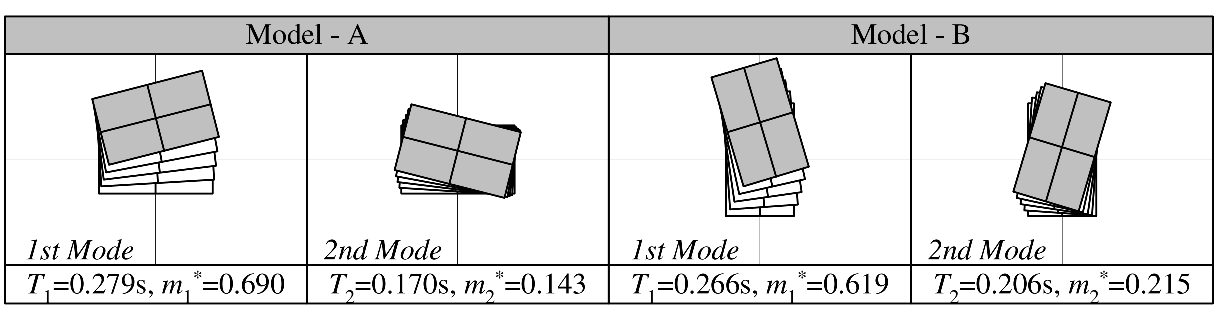


Figure 8 Mode Shapes of Building Models

Table 2 Input Ground Motion

Ground Motion Record ID	Artificial Ground Motion ID	Max. Ground Acceleration (m/s^2)
ELC	JCode-ELC	5.703
TAF	JCode-TAF	6.064
HAC	JCode-HAC	5.210
TOH	JCode-TOH	5.676
KMO	JCode-KMO	6.507
FKI	JCode-FKI	5.892

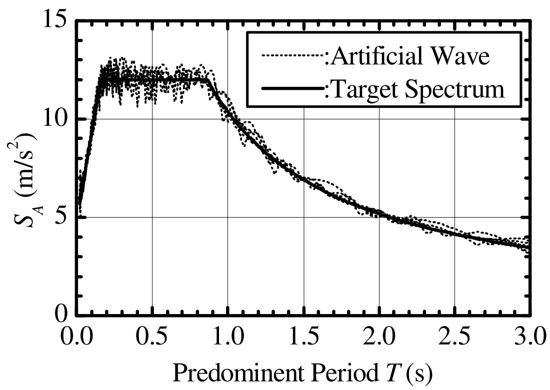


Figure 9 Elastic Acceleration Spectra

models m_1^* are larger than 0.6.

Ground Motion Data

In this study, the earthquake excitation is considered unidirectional in Y-direction, and six artificial ground motions are used. Target elastic spectrum with 5% of critical damping $S_A(T, 0.05)$ is determined by Eq. (41).

$$S_A(T, 0.05) = \begin{cases} 4.8 + 45T & & T < 0.16s \\ 12.0 & \text{m/s}^2 & 0.16s \le T < 0.864s \\ 12.0 \cdot (0.864/T) & & T \ge 0.864s \end{cases} \tag{41}$$

Where T is the natural period of the SDOF model. The first 40.96 seconds (2^{12} = 4096 data, 0.01 second sampling) of following record are used to determine phase angles of the ground motion: the NS component of El Centro 1940 record (referred to as ELC), NS component of Taft 1952 record (TAF), EW component of Hachinohe 1968 record (HAC), NS component of Tohoku University 1978 record (TOH), NS component of Kobe Meteorological Observatory 1995 (KMO) and Fukiai 1995 (FKI) record. Table 2 shows the maximum acceleration of artificial ground motion and Figure 9 shows the elastic acceleration response spectra of artificial ground motion with 5% of critical damping.

Numerical Time-history Analysis Procedures

In this study, results of proposed procedure are compared with those obtained by nonlinear dynamic time-history analysis. In the nonlinear dynamic time-history analysis, the damping matrix is assumed proportional to the instant stiffness matrix and 3% of the critical damping for the first mode. The Newmark-β method (β = 1/4) is applied in numerical integrations.

Analysis Results and Discussions

Figure 10 shows the comparisons of the maximum displacement at each frame on top floor, and Figure 11 shows the comparisons of the maximum drift angle at each frame, obtained from time-history analysis of multi-story asymmetric building model (mean value of the 6 analysis μ, and mean $\pm$ standard deviation $\mu \pm \sigma$ are shown) and proposed procedure. These figure shows that the proposed procedure can estimate the response of multi-story asymmetric building model satisfactory.

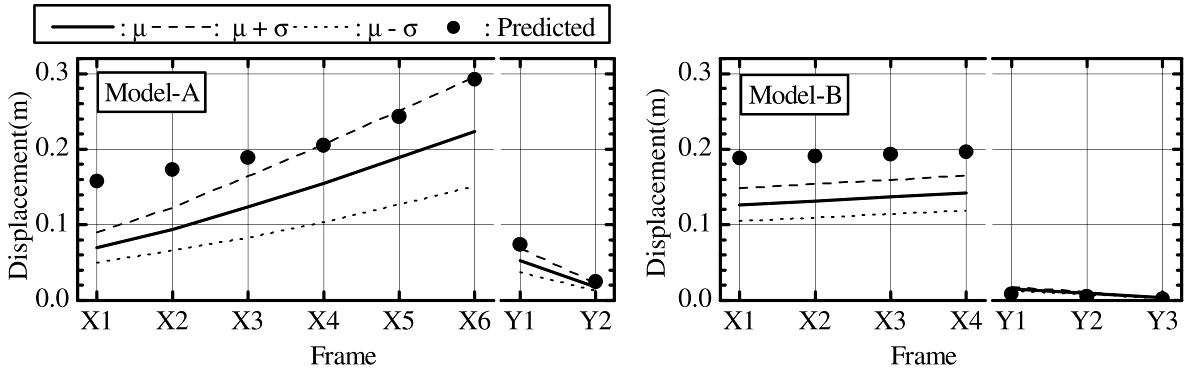


Figure 10 Prediction of Maximum Displacement at Each Frame on Top Floor

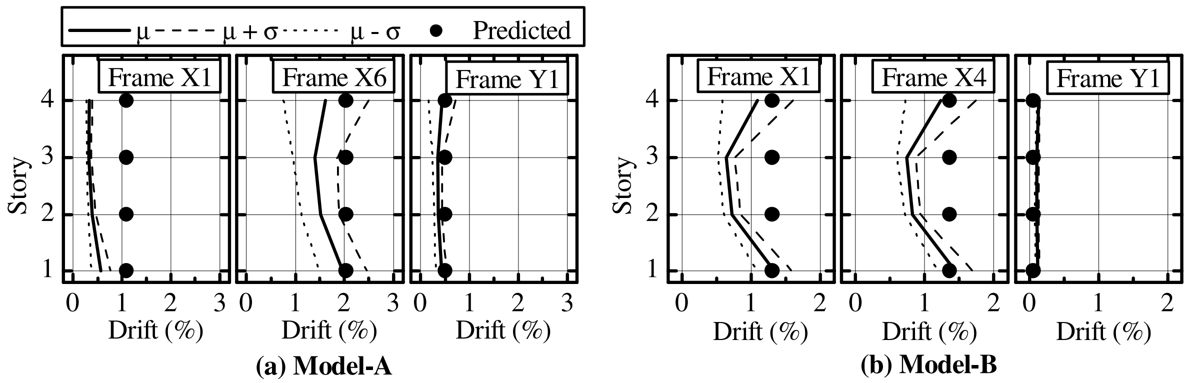


Figure 11 Prediction of Maximum Drift Angle at Each Frame

CONCLUSIONS

In this paper, a simplified procedure for multi-story asymmetric buildings subjected to unidirectional ground motion is proposed, and the results obtained by the proposed procedure are compared with the results obtained by nonlinear dynamic time-history analysis. The results show that the nonlinear analysis of multi-story asymmetric buildings subjected to unidirectional ground motion can be satisfactorily estimated by the simplified procedure proposed in this study.

APPENDIX. PUSHOVER ANALYSIS PROCEDURE CONSIDERING THE CHANGE IN THE FIRST MODE SHAPE AT EACH NONLINEAR STAGE

The pushover analysis procedure of the equivalent single-story model considering the change in the first mode shape at each nonlinear stage is summarized below in a step-by-step form:

1) Set a displacement increment at the next step ${}_{n+1}\Delta Y_1^*$.
2) Determine the first mode vector at the current step ${}_n\Gamma_{T1}\{{}_n\phi_{T1}\}$.
3) Assume the first mode vector at the next step as ${}_{n+1}\Gamma_1\{{}_{n+1}\phi_{T1}\}={}_n\Gamma_1\{{}_n\phi_{T1}\}$.
4) Calculate the displacement by Eq. (A1):

$$\{{}_{n+1}d\}={}_{n+1}\Gamma_{T1}\{{}_{n+1}\phi_{T1}\}{}_{n+1}D_1^* \quad \text{(A1)}$$

where $${}_{n+1}D_1^* = \frac{1}{{}_{n+1}\Gamma_{T1}}\left({}_nY_1^* + {}_{n+1}\Delta Y_1^*\right) \quad \text{(A2)}$$

5) Determine the deformation and restoring force of each element.
6) Determine the first mode vector ${}_{n+1}\Gamma_{T1}\{{}_{n+1}\phi_{T1}\}$ from the secant stiffness defined at the maximum deformation previously experienced.
7) Repeat steps 4) through 6) until ${}_{n+1}\Gamma_{T1}\{{}_{n+1}\phi_{T1}\}$ calculated in step 6) falls within an allowable band from the assumed first mode shape.

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